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Parametric Identification of Systems with General Backlash

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Abstract. The notion of general backlash is introduced where instead of the straight lines determining the upward and downward parts of backlash characteristic general curves are considered. An analytic form of general backlash characteristic description is proposed, which is based on appropriate switching and internal functions. Consequently, this multi-valued mapping is represented by one difference equation. All the parameters in the equation describing this hard nonlinearity are separated; hence the general backlash identification can be solved as a quasi-linear problem using an iterative parameter estimation method with internal variable estimation. Also the identification of cascaded systems consisting of a general input backlash followed by a linear dynamic system is presented. Simulation studies of general backlash identification and that of cascaded systems with general input backlash are included.

Keywords: backlash, general backlash, modeling, identification.

1. Introduction

One of the most important nonlinearities that limit control systems performance in many applications is the so-called backlash (Kalaš *et al.*, 1985). Backlash appears whenever two physical parts are supposed to move together and there is an amount of space between the parts. Some systems must have some amount of backlash in order to function, for example a gear box needs some space for heat expansion.

The backlash can be classified as a hard (i.e., nondifferentiable) and dynamic nonlinearity. The presence of such nonlinearities influences the performance by affecting static accuracy of the control systems. This kind of nonlinearity may often cause delays, oscillations and inaccuracy which severely limit the performance of control systems and compensation of backlash has attracted research effort by several decades (Nordin and Gutman, 2002; Tao and Canudas de Wit, 1997; Tao and Kokotovic, 1993). Actuator and sensor nonlinearities are among the key factors limiting both static and dynamic performance of feedback control systems. For example, backlash in gears and other mechanical components prevents accurate positioning and may lead to chattering and limit-cycle instabilities. This in turn increases backlash. In general, backlash could be present at the input of the system, output of the system, or at both the input and the output.

However, in most applications the backlash parameters are either poorly known or completely unknown, hence the identification of backlash is fundamental for its compensation and implementation of the corresponding inverse. Unfortunately, there are only few contributions in the literature on the identification of systems with nonstatic hard nonlinearities, e.g., (Bai, 2002; Vörös, 1997) and even fewer on backlash identification (Cerone and Regruto, 2007; Dong *et al.*, 2009, 2010; Giri *et al.*, 2008a; Sun *et al.*, 1999). Moreover, it is assumed that the backlash is "ideal", i.e., straight lines approximate the upward and downward curves of the characteristic. This simplifies the system description, however, in some cases it may lead to inaccuracies.

The components of control systems may be free from backlash when new, but after some time in use the wear results in an introduction of backlash in the systems. In general the form of backlash changes with time and wear, regardless of what form of backlash was present when the component was new. Therefore it may be appropriate to generalize the backlash and consider general upward and downward curves.

The only works dealing with the identification of Hammerstein-like systems with general switch and backlash nonlinearities were published in Giri *et al.* (2008b), Rochdi *et al.* (2010). The proposed identification method consists of two independent, but structurally symmetric, identification schemes. The first one determines the points located on the descendent border of general nonlinearity as well as the parameters of the linear subsystem. The second identification scheme determines the points located on the ascendent border of general nonlinearity and the parameters of the linear subsystem. The key idea is to use pulse-type periodic input signals so that only the points of interest are excited on each border.

In this paper, an alternative approach to the identification of systems with general backlash is described. A simple identification method based on a new mathematical model for general backlash is proposed. First, an analytic description of this hard dynamic nonlinearity is introduced based on a compound operator decomposition approach, which uses appropriate switching functions and internal variables and is a generalization of the backlash description presented in Vörös (2010a). The general backlash parameters in the resulting model equation are separated; hence their estimation is solved as a quasi-linear problem using an iterative method with internal variable estimation similarly as in (Vörös, 1999, 2002). Then the identification of cascade systems consisting of a general input backlash followed by a linear dynamic system is presented. In contrast to the well-known Hammerstein systems, a *nonlinear dynamic system* and a *linear dynamic system* are cascaded in this case. Application of the decomposition technique leads to a system description, which is again quasi-linear, and the parameters of cascade system are estimated iteratively based on available inputs and outputs. Simulation studies of general backlash identification and that of cascaded systems with general input backlash illustrate the feasibility of proposed identification methods.

2. Ideal Backlash

The discrete-time mathematical description for the ideal backlash nonlinearity with inputs $u(t)$ and outputs $x(t)$ shown in Fig. 1, is given by Cerone and Regruto (2007), Tao and

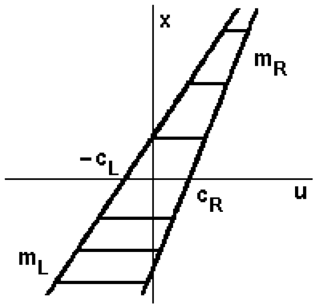

Fig. 1. Ideal backlash characteristic.

Kokotovic (1993) as

$$x(t) = \begin{cases} m_L[u(t) + c_L], & u(t) \leqslant z_L, \\ x(t-1), & z_L \leqslant u(t) \leqslant z_R, \\ m_R[u(t) - c_R], & u(t) \geqslant z_R, \end{cases} \tag{1}$$

with z_L and z_R defined by

$$x(t-1) = m_L(z_L + c_L), \tag{2}$$

$$x(t-1) = m_R(z_R - c_R), \tag{3}$$

where the slopes $m_L,\ m_R$, the dead zone constants $c_L > 0,\ c_R > 0$ characterize the backlash and z_L and z_R are the u-axis values of intersections of the two lines with the horizontal inner segment containing $x(t-1)$.

A special form of backlash description was proposed in Vörös (2010) to specify the three branches of (1) in one equation. This is based on the function

$$h(s) = \begin{cases} 0, & \text{if } s > 0, \\ 1, & \text{if } s \leqslant 0, \end{cases} \tag{4}$$

switching between two sets of values, i.e., $(-\infty, s)$ and (s, ∞), and the complementary function to $h(s)$, that is $[1 - h(s)]$. Defining the following variables based on (2) and (3):

$$f_1(t) = h\big[u(t) - z_L\big] = h\big\{\big[m_L u(t) + m_L c_L - x(t-1)\big]/m_L\big\}, \tag{5}$$

$$f_2(t) = h\big[z_R - u(t)\big] = h\big\{\big[x(t-1) - m_R u(t) + m_R c_R\big]/m_R\big\}, \tag{6}$$

the backlash, which is a multi-valued mapping, can be described by one difference equation as:

$$\begin{aligned} x(t) = {} & m_L u(t) f_1(t) + m_L c_L f_1(t) + m_R u(t) f_2(t) - m_R c_R f_2(t) \\ & + x(t-1)\big[1 - f_1(t)\big]\big[1 - f_2(t)\big]. \end{aligned} \tag{7}$$

The input/output relation (7) is identical with that of (1). The slopes of straight lines m_L and m_R may be simultaneously positive or negative, while the constants c_L and c_R, determining the dead zone, must be positive. This equation allows the upward and downward line slopes to be different provided that the intersection of the two lines is not in the region of practical interest.

3. General Backlash

(a) Description

In the above mentioned case of ideal backlash the left and right branches of the characteristic are considered to be straight lines. However, in some applications the straight lines are only advantageous approximations of general curves constituting the left and right branches of backlash as shown in Fig. 2. Therefore the backlash can be generalized in the following way.

The general backlash characteristic can be described by the equation

$$x(t) = \begin{cases} L[u(t)], & u(t) \leqslant z_L, \\ x(t-1), & z_L \leqslant u(t) \leqslant z_R, \\ R[u(t)], & u(t) \geqslant z_R, \end{cases} \tag{8}$$

where the mappings $L[u(t)]$ and $R[u(t)]$ describe the left and right branches of the characteristic, respectively, the u-axis values z_L and z_R, by analogy with (2) and (3), are given as follows:

$$x(t-1) = L(z_L), \tag{9}$$

$$x(t-1) = R(z_R). \tag{10}$$

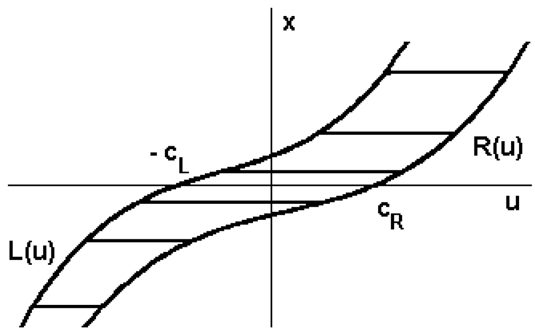

Fig. 2. General backlash characteristic.

Assume the left and right curves can be approximated by the polynomials

$$L\big[u(t)\big] = \sum_{i=1}^{n} m_{Li}\big[u(t) + c_L\big]^i, \tag{11}$$

$$R\big[u(t)\big] = \sum_{i=1}^{n} m_{Ri}\big[u(t) - c_R\big]^i, \tag{12}$$

respectively, where $c_L > 0,\ c_R > 0$ are the intersections of $L[u(t)]$ and $R[u(t)]$ with the u-axis. Then the general backlash characteristic can be written as

$$x(t) = \begin{cases} \sum_{i=1}^{n} m_{Li}\big[u(t) + c_L\big]^i, & u(t) \leqslant z_L, \\ x(t-1), & z_L \leqslant u(t) \leqslant z_R, \\ \sum_{i=1}^{n} m_{Ri}\big[u(t) - c_R\big]^i, & u(t) \geqslant z_R, \end{cases} \tag{13}$$

where

$$x(t-1) = \sum_{i=1}^{n} m_{Li}[z_L + c_L]^i, \tag{14}$$

$$x(t-1) = \sum_{i=1}^{n} m_{Ri}[z_R - c_R]^i. \tag{15}$$

Now, an analogous approach, as was done in the previous Section, can be applied to the description of general backlash. After introducing the internal variables

$$\xi_1(t) = u(t) + c_L, \tag{16}$$

$$\xi_2(t) = u(t) - c_R, \tag{17}$$

the following variables based on (14) and (15) can be defined:

$$f_1(t) = h\left[\sum_{i=1}^{n} m_{Li}\xi_1^i(t) - x(t-1)\right], \tag{18}$$

$$f_2(t) = h\left[x(t-1) - \sum_{i=1}^{n} m_{Ri}\xi_2^i(t)\right]. \tag{19}$$

Then the general backlash can be described by one difference equation as follows:

$$\begin{aligned} x(t) = {} & \sum_{i=1}^{n} m_{Li}\xi_1^i(t) f_1(t) + \sum_{i=1}^{n} m_{Ri}\xi_2^i(t) f_2(t) \\ & + x(t-1)\big[1 - f_1(t)\big]\big[1 - f_2(t)\big]. \end{aligned} \tag{20}$$

To include the dead zone parameters c_L and c_R into the backlash description, we can separate the first terms of the sums in (20) and half-substitute from (16) and (17) as follows:

$$x(t) = m_{L1}u(t)f_1(t) + m_{L1}c_L f_1(t) + \sum_{i=2}^{n} m_{Li}\xi_1^i(t)f_1(t) + m_{R1}u(t)f_2(t)$$
$$- m_{R1}c_R f_2(t) + \sum_{i=2}^{n} m_{Ri}\xi_2^i(t)f_2(t) + x(t-1)\big[1 - f_1(t)\big]\big[1 - f_2(t)\big]. \tag{21}$$

Now the input/output relation for the generalized backlash (21) is identical with that of (8). All the parameters are separated and the equation is linear in the input, output and internal variables. This description allows the upward and downward curves to be different provided that the intersection of the two curves is not in the region of practical interest.

(b) Parameter Estimation

The proposed new description can be used for estimation of generalized backlash parameters. Defining the following vector of data

$$\varphi(t) = \big[u(t)f_1(t), f_1(t), \xi_1^2(t)f_1(t), \ldots, \xi_1^n(t)f_1(t),$$
$$u(t)f_2(t), -f_2(t), \xi_2^2(t)f_2(t), \ldots, \xi_2^n(t)f_2(t)\big]^T, \tag{22}$$

and the vector of parameters

$$\theta = [m_{L1}, m_{L1}c_L, m_{L2}, \ldots, m_{Ln}, m_{R1}, m_{R1}c_R, m_{R2}, \ldots, m_{Rn}]^T, \tag{23}$$

the mathematical model for generalized backlash can be written in the vector form

$$x(t) - x(t-1)\big[1 - f_1(t)\big]\big[1 - f_2(t)\big] = \varphi^T(t)\theta + e(t), \tag{24}$$

where $e(t)$ is an additive noise.

As the variables $\xi_1(t),\ \xi_2(t),\ f_1(t)$ and $f_2(t)$ in (22) are unmeasurable and must be estimated, an iterative parameter estimation process has to be considered, similarly as in Vörös (1999, 2002). Assigning the estimated variables in the sth step as

$${}^s\xi_1(t) = u(t) + {}^s c_L, \tag{25}$$

$${}^s\xi_2(t) = u(t) - {}^s c_R, \tag{26}$$

$${}^s f_1(t) = h\left[\sum_{i=1}^{n} {}^s m_{Li} {}^s\xi_1^i(t) - x(t-1)\right], \tag{27}$$

$${}^s f_2(t) = h\left[x(t-1) - \sum_{i=1}^{n} {}^s m_{Ri} {}^s\xi_2^i(t)\right], \tag{28}$$

the error to be minimized in the estimation procedure is

$$^{s+1}\varepsilon(t) = x(t) - x(t-1)\big[1 - {}^{s}f_1(t)\big]\big[1 - {}^{s}f_2(t)\big] - {}^{s}\varphi^T(t)^{s+1}\theta, \tag{29}$$

where ${}^{s}\varphi(t)$ is the data vector with the corresponding estimates of variables $\xi_1(t),\ \xi_2(t),$ $f_1(t)$ and $f_2(t)$ according to (25)–(28) and ${}^{s+1}\theta$ is the $(s+1)$th estimate of the parameter vector.

The steps in the iterative procedure may be now stated as follows:

(a) Minimizing the least squares criterion based on (29) for $t = 1, 2, \ldots, N$ samples of inputs and outputs, the estimates of parameters ${}^{s+1}\theta$ are computed using ${}^{s}\varphi(t)$ with the sth estimates of variables, i.e., ${}^{s}\xi_1(t),\ {}^{s}\xi_2(t),\ {}^{s}f_1(t)$ and ${}^{s}f_2(t)$.

(b) Using (25)–(28) the estimates of ${}^{s+1}\varphi(t)$ are evaluated by means of the recent estimates of corresponding parameters and variables.

(c) If the estimation criterion is met the procedure ends, else it continues by repeating steps (a) and (b).

In the first iteration nonzero initial values of the parameters $m_{L1},\ m_{R1},\ c_L$ and c_R have to be considered for evaluation of ${}^{1}\varphi(t)$ to start up the iterative algorithm. In the simplest case ${}^{1}m_{L1} = {}^{1}m_{R1} = 1$ for the general backlash with increasing curves or ${}^{1}m_{L1} = {}^{1}m_{R1} = -1$ for the general backlash with decreasing curves, while c_L and c_R are chosen small enough.

The key properties of the proposed algorithm (convergence, bias, consistency) can be considered as analogous to those of the applied least-squares algorithm, because always the corresponding polynomial segment of the nonlinearity is included into the computation. However, only the estimates of internal variables are used in the data vector, which depend on the previous estimates of corresponding parameters and variables. Therefore the convergence of the above algorithm with estimation of internal variables cannot be exactly proved.

(c) Simulation Studies

The method for the identification of general backlash was implemented and tested in MATLAB. Several cases were simulated and the estimations of parameters were carried out on the basis of input and output records. The performance of the proposed method is illustrated on the following examples.

EXAMPLE 1. The backlash shown in Fig. 3 was simulated with the following parameters: $m_{L1} = 0.5,\ m_{L2} = -0.3,\ m_{L3} = 0.3,\ c_L = 0.7,\ m_{R1} = 0.6,\ m_{R2} = 0.4,\ m_{R3} = 0.2,\ c_R = 0.8$. The identification was performed on the basis of $N = 800$ samples of uniformly distributed random inputs with $|u(t)| < 2.0$ and simulated outputs. Normally distributed random noise with zero mean and signal-to-noise ratio – $\mathrm{SNR} = 25$ (the square root of the ratio of output and noise variances) was added to the outputs. The iterative estimation algorithm was applied with initial values ${}^{1}m_{L1} = {}^{1}m_{R1} = 1$ and ${}^{1}c_L = {}^{1}c_R = 0.001$ for the first estimates of $\xi_1(t),\ \xi_2(t),\ f_1(t)$ and $f_2(t)$. The process of parameter estimation is shown in Fig. 4 (the top-down order of parameters is

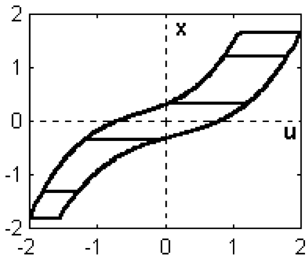

Fig. 3. General backlash characteristic – Example 1.

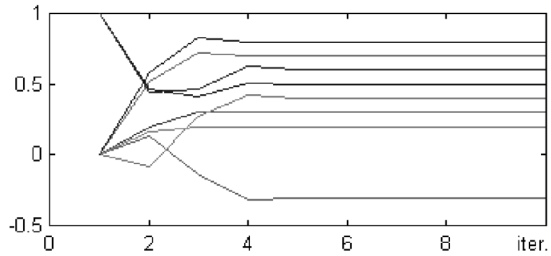

Fig. 4. Parameter estimates – Example 1.

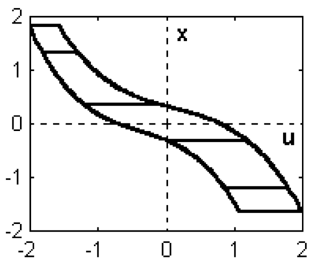

Fig. 5. General backlash characteristic – Example 2.

$c_R, c_L, m_{R1}, m_{L1}, m_{R2}, m_{L3}, m_{R3}, m_{L2})$. The estimates converge to the values of real parameters after 5 iterations.

EXAMPLE 2. The general backlash (with decreasing both the left and the right curves) shown in Fig. 5 was simulated with the following parameters: $m_{L1} = -0.5$, $m_{L2} = 0.3$, $m_{L3} = -0.3$, $c_L = 0.7$, $m_{R1} = -0.6$, $m_{R2} = -0.4$, $m_{R3} = -0.2$, $c_R = 0.8$. The identification was performed under the same conditions as in Example 1 only the initial values ${}^1m_{L1} = {}^1m_{R1} = -1$. The process of parameter estimation is shown in Fig. 6 (the top-down order of parameters is c_R, c_L, m_{L2}, m_{R3}, m_{L3}, m_{R2}, m_{L1}, m_{R1}). The estimates converge to the values of real parameters after 5 iterations.

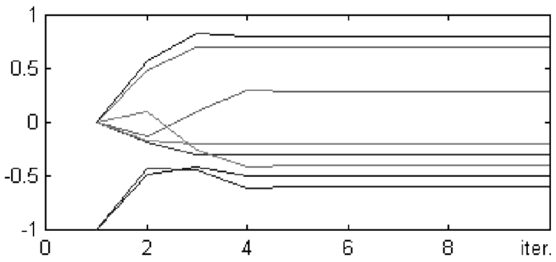

Fig. 6. Parameter estimates – Example 2.

As the simulation studies show the convergence of the proposed identification algorithm is good despite the relatively high level of additive noise. This is because the nonlinearity has actually simple polynomial segments and the switching functions separate these segments. Hence the main estimation problem is with the "hard nonlinear element" of the general backlash, i.e., the unknown deadzones.

4. Cascade System with General Input Backlash

(a) Description

Cascade systems consist of serially connected linear and nonlinear subsystems. One of the simplest cases is the connection of a static nonlinear subsystem followed by a linear dynamic one. This cascade system is known as the Hammerstein system and there are lots of identification methods for different types of nonlinearities and corresponding models, e.g., Bai (2003), Bai and Li (2004). Bai *et al.* (2007), Bako *et al.* (2009), Chen (2009), Chen *et al.* (2009), Ding *et al.* (2006, 2011), Dolanc and Strmcnik (2005), Giri *et al.* (2001), Hasiewicz and Mzyk (2004), Hasiewicz *et al.* (2005), Janczak (2003, 2005), Lacy and Bernstein (2005), Liu and Bai (2007), Mzyk (2007), Pupeikis (2005, 2006, 2010), Sliwinski *et al.* (2009), Szabo *et al.* (2010), Wang and Ding (2011), Wang *et al.* (2008), Zhang and Tan (2008).

In many real control systems the backlash appears in a cascade connection with a linear dynamic system. One of the possible cases is the cascade system where the general backlash is followed by a linear dynamic system as shown in Fig. 7. Compared to the Hammerstein systems, the essential difference is that a nonlinear dynamic system and a linear dynamic system are cascaded in this case.

The linear dynamic system can be described by the difference equation

$$y(t) = \sum_{i=1}^{r} a_i x(t-i) - \sum_{j=1}^{p} b_j y(t-j), \tag{30}$$

where $x(t)$ and $y(t)$ are the inputs and outputs, respectively. The nonlinear block consists of a general backlash.

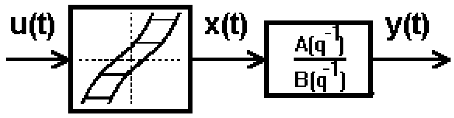

Fig. 7. Cascade systems with general input backlash.

The output equation of this cascade system can be constructed from (21) and (30). However, a direct substitution of (21) into (30) would lead to a very complex expression, therefore the so-called key term separation principle can be applied (Vörös, 1999). In the connection of these subsystems, if the first subsystem is multiplied by a nonzero real constant and if the second one is divided by the same constant, the resulting cascade system has the same input-output behavior. Therefore we can assume that $a_1 = 1$ and substitute (21) into (30) only for the separated variable $x(t-1)$ leading to the following equation

$$\begin{aligned} y(t) = {} & m_{L1}u(t-1)f_1(t-1) + m_{L1}c_L f_1(t-1) \\ & + \sum_{i=2}^{n} m_{Li}\xi_1^i(t-1)f_1(t-1) + m_{R1}u(t-1)f_2(t-1) - m_{R1}c_R f_2(t-1) \\ & + \sum_{i=2}^{n} m_{Ri}\xi_2^i(t-1)f_2(t-1) + x(t-2)\big[1-f_1(t-1)\big]\big[1-f_2(t-1)\big] \\ & + \sum_{i=2}^{r} a_i x(t-i) - \sum_{j=1}^{p} b_j y(t-j), \end{aligned} \tag{31}$$

where the parameters of both the general backlash and the linear system are separated and the equation is quasi-linear as the variables $\xi_1(t),\ \xi_2(t),\ f_1(t)$ and $f_2(t)$ depend on the backlash parameters.

(b) Parameter Estimation

The parameter estimation for the cascade system with general input backlash can be performed similarly, as in the previous case. Defining the vector of data

$$\begin{aligned} \Phi(t) = {} & \big[u(t-1)f_1(t-1), f_1(t-1), \xi_1^2(t-1)f_1(t-1), \dots, \\ & \xi_1^n(t-1)f_1(t-1), u(t-1)f_2(t-1), -f_2(t-1), \\ & \xi_2^2(t-1)f_2(t-1), \dots, \xi_2^n(t-1)f_2(t-1), x(t-2), \dots, \\ & x(t-r), -y(t-1), \dots, -y(t-p)\big]^T, \end{aligned} \tag{32}$$

and the vector of parameters

$$\begin{aligned} \Theta = {} & [m_{L1}, c_1, m_{L2}, \dots, m_{Ln}, m_{R1}, c_2, m_{R2}, \dots, m_{Rn}, a_2, \dots, \\ & a_r, b_1, \dots, b_p]^T, \end{aligned} \tag{33}$$

i.e.,

$$c_L = c_1/m_{L1}, \qquad c_R = c_2/m_{R1}, \tag{34}$$

the mathematical model for the cascase system with general input backlash can be written in the vector form

$$y(t) - x(t-2)\big[1 - f_1(t-1)\big]\big[1 - f_2(t-1)\big] = \Phi^T(t)\Theta + e(t). \tag{35}$$

where $e(t)$ is an additive noise.

As the variables $\xi_1(t),\ \xi_2(t),\ f_1(t),\ f_2(t)$ and $x(t)$ in (32) are unmeasurable and must be estimated, again an iterative parameter estimation process has to be considered. Assigning the estimates of variables $\xi_1(t),\ \xi_2(t),\ f_1(t),\ f_2(t)$ in the sth step as (25)–(28) and the estimates of internal variable $x(t)$ as

$$\begin{aligned} {}^s x(t) &= {}^s m_{L1} u(t) {}^s f_1(t) + {}^s m_{L1} {}^s c_L {}^s f_1(t) \\ &\quad + \sum_{i=2}^{n} {}^s m_{Li} {}^s \xi_1^i(t) {}^s f_1(t) + {}^s m_{R1} u(t) {}^s f_2(t) - {}^s m_{R1} {}^s c_R {}^s f_2(t) \\ &\quad + \sum_{i=2}^{n} {}^s m_{Ri} {}^s \xi_2^i(t) {}^s f_2(t) + {}^s x(t-1)\big[1 - {}^s f_1(t)\big]\big[1 - {}^s f_2(t)\big], \end{aligned} \tag{36}$$

the error to be minimized in the estimation procedure is

$$^{s+1}\varepsilon(t) = y(t) - {}^s x(t-2)\big[1 - {}^s f_1(t-1)\big]\big[1 - {}^s f_2(t-1)\big] - {}^s\Phi^T(t) {}^{s+1}\Theta, \tag{37}$$

where ${}^s\Phi(t)$ is the data vector with the corresponding estimates of variables $\xi_1(t),\ \xi_2(t),\ f_1(t),\ f_2(t)$ and $x(t)$ according to (25)–(28) and (36) and ${}^{s+1}\Theta$ is the $(s+1)$th estimate of the parameter vector.

The steps in the iterative procedure may be now stated as follows:

(a) Minimizing the least squares criterion based on (37) for N samples of inputs and outputs, the estimates of parameters ${}^{s+1}\Theta$ are computed using ${}^s\Phi(t)$ with the sth estimates of variables ${}^s\xi_1(t), {}^s\xi_2(t), {}^s f_1(t)$ and ${}^s f_2(t)$.

(b) Using (25)–(28), however with estimates of internal variable $x(.)$, and (36) the estimates of ${}^{s+1}\Phi(t)$ are evaluated by means of the recent estimates of corresponding parameters and variables.

(c) If the estimation criterion is met the procedure ends, else it continues by repeating steps (a) and (b).

In the first iteration, equally as in the previous Section, nonzero initial values of the general backlash parameters $m_{L1},\ m_{R1},\ c_L$ and c_R have to be considered for evaluation of ${}^1\Phi(t)$ to start up the iterative algorithm, while the initial values of linear system parameters can be chosen zero.

(c) Simulation Studies

Several cases of the cascade systems with generalized input backlash were simulated and the estimations of parameters were carried out on the basis of input and output records. The performance of the proposed method is illustrated on the following examples.

EXAMPLE 3. The cascade system with general input backlash (Fig. 8) characterized by the parameters $m_{L1} = 0.5,\ m_{L2} = -0.3,\ m_{L3} = 0.3,\ c_L = 0.45,\ m_{R1} = 0.6,\ m_{R2} = 0.4,\ m_{R3} = 0.2,\ c_R = 0.55$ and followed by the linear dynamic system described by the difference equation

$$y(t) = x(t-1) + 0.15x(t-2) + 0.2y(t-1) - 0.35y(t-2)$$

was considered. The identification was performed on the basis of $N = 1500$ samples of uniformly distributed random inputs with $|u(t)| < 1.0$ and simulated outputs. Normally distributed random noise with zero mean and SNR = 50 was added to the outputs. The iterative estimation algorithm was applied with initial values ${}^1m_{L1} = {}^1m_{R1} = 1$ and ${}^1c_L = {}^1c_R = 0.001$ for the first estimate of $\xi_1(t),\ \xi_2(t),\ f_1(t)$ and $f_2(t)$, while the initial values of linear system parameters were chosen zero. The process of backlash parameter estimation is shown in Fig. 9 (the top-down order of parameters is $m_{R1},\ c_R,\ m_{L1},\ c_L,\ m_{R2},\ m_{L3},\ m_{R3},\ m_{L2}$) and the process of linear block parameter estimation is shown in Fig. 10. The estimates converge to the values of real parameters after 8 iterations.

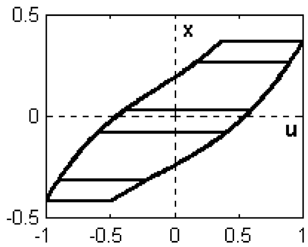

Fig. 8. General backlash characteristic – Example 3.

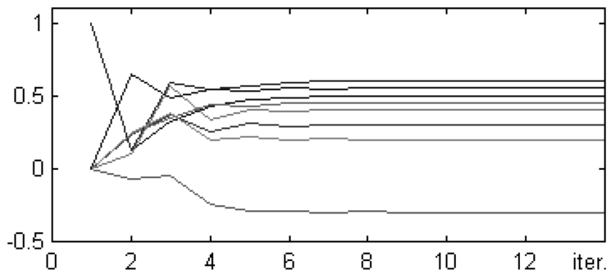

Fig. 9. Backlash parameter estimates – Example 3.

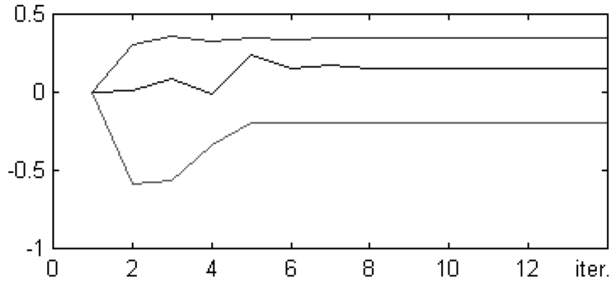

Fig. 10. Linear block parameter estimates – Example 3.

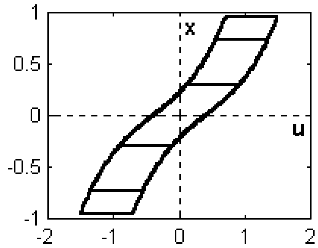

Fig. 11. General backlash characteristic – Example 4.

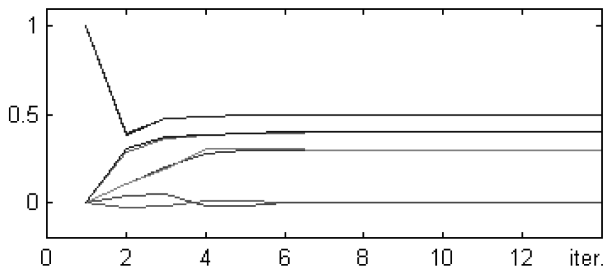

Fig. 12. Backlash parameter estimates – Example 4.

EXAMPLE 4. The cascade system with "equidistant" general input backlash (Fig. 11) characterized by the parameters $m_{L1} = 0.5,\ m_{L2} = 0.0,\ m_{L3} = 0.3,\ c_L = 0.4,\ m_{R1} = 0.5,\ m_{R2} = 0.0,\ m_{R3} = 0.3,\ c_R = 0.4$ and followed by the same linear dynamic system as above was considered. The identification was performed on the same basis as in Example 3 with $|u(t)| < 1.5$. The backlash parameter estimates are shown in Fig. 12 (the top-down order of parameters is $m_{R1} = m_{L1},\ c_R = c_L,\ m_{R3} = m_{L3},\ m_{R2} = m_{L2}$) and those of linear block are shown in Fig. 13. The estimates converge to the values of real parameters after 7 iterations.

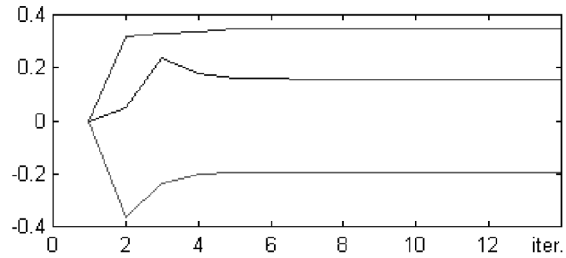

Fig. 13. Linear block parameter estimates – Example 4.

5. Conclusions

In this paper a new analytic form of general backlash characteristic was introduced, where this three-branch nonlinearity is described by one output equation with separated parameters. The new model was applied to the identification of general backlash systems and the cascaded systems consisting of a general input backlash followed by a linear dynamic system. For both cases, iterative parameter estimation algorithms were proposed and their feasibility was shown in simulation studies. Although no convergence proof of the identification methods with internal variable estimation is available, testing of the proposed algorithms showed very good results. Compared to Giri *et al.* (2008b), Rochdi *et al.* (2010), the proposed identification methods do not require special input signals, however, they are not dealing with dynamic nonlinearities of the switch type.

Finally note, that the presented model of general backlash and the parameter estimation method can be easily extended for other types of cascaded systems (Vörös, 2010b).

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Sistemų su paprastąja rimties eiga parametrinis identifikavimas

Jozef VÖRÖS

Supažindinama su paprastąja rimties eiga, kurioje vietoje aukštyn bei žemyn nukreiptųjų tiesių, apibrėžiančių rimties eigos charakteristikos dalis, taikomos atitinkamos kreivės. Darbe pasiūlyta analitinė forma paprastosios rimties eigos charakteristikos aprašymui, kuri grindžiama tarpusavyje susijusiomis perjungimo bei vidinėmis funkcijomis. Todėl šis daugiareikšmis atvaizdavimas pateikiamas vienintele skirtumine lygtimi. Visi šios lygties, aprašančios „kietąjį“ netiesiškumą, parametrai esti atskirti; taigi paprastosios rimties eigos identifikavimas gali būti sprendžiamas kaip kvazi-tiesinis uždavinys, taikant iteracinį parametrų identifikavimo metodą su vidinio kintamojo įvertinimu. Darbe taip pat nagrinėjamas kaskadinių sistemų, susidedančių iš paprastosios rimties eigos ir po jos sekančios tiesinės dinaminės sistemos, identifikavimo uždavinys. Pateikti rimties eigos bei kaskadinių sistemų su paprastąja rimties eiga modeliavimo ir jų identifikavimo rezultatai.