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A Forward–Backward Approach for Instantaneous Frequency Estimation of Frequency Modulated Signals in Noisy Environment

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Abstract. In this paper a forward–backward basis function approach for instantaneous frequency estimation of the frequency-modulated signal in noisy environment is presented. At first, a forward–backward prediction approach is applied for least squares estimation of time-varying autoregressive parameters. A time-varying parameters are expressed as a summation of constants multiplied by basis functions. Then, the time-varying frequencies are extracted from the time-varying autoregressive parameters by calculating the angles of the estimation error filter polynomial roots. The experimental results are presented, which shows the superiority of the proposed method against the covariance (forward prediction) approach.

Keywords: instantaneous frequency estimation, forward–backward approach, basis functions, noisy environment.

1. Introduction

Frequency-modulated (FM) signal estimation in a noisy environment is important for many commercial and military applications. These signals are analyzed in engineering applications such as telecommunications, biomedical engineering, radar, sonar, and signal processing. The instantaneous frequency estimation is based on system identification methods (Atanosov and Ichtev, 2011). The instantaneous frequency (IF) characterizes important properties of the signal. The concept of IF estimation was reviewed in Boashash (1991). Several methods have been proposed for this task. One approach is based on the wavelet shrinkage technique (Donoho *et al*., 1995), which denoises data in the wavelet domain. Another approach is to represent the data by using a time-frequency Gabor transform (Qian, 1993). However, these methods works poorly when signal to noise ratio (SNR) is lower than 0 dB. IF estimation methods have been developed for multicomponent signals embedded in additive noise (Peleg, 1996). Parametric methods use a polynomial phase representation of the frequency modulated signals for IF estimation. In Hussain *et. al*. (2002) a method for IF estimation is based on quadratic time-frequency distributions of the signal. An iterative algorithm using the cross Wignes–Ville distribution (WVD) is presented in Boashash *et al.* (1993). An adaptive algorithm for IF estimation using WVD was presented in Katkovnik (1998). The IF estimate depends on the fact that the WVD has a maximum around the IF law of the signal. A parametric approach for IF estimation of multicomponent signal was presented in Francos (1999). However, the parametric approaches in time-frequency analysis suffer from the computational complexity (Chi, 1989). For a multicomponent time-varying harmonic model, particle filtering approaches have been analyzed in Dubois (2005), Tsakonas *et al.* (2008). In Dubois (2005), a Gaussian random walk model is employed for the evoliution of the parameters. In Tsakonas *et al.* (2008), the problem of tracking the time-varying parameters of a harmonic or chirp signal using particle filtering tools is considered. It is shown, that the optimal importance function that minimizes the variance of the particle weights can be computed in closed form.

In this paper a new forward–backward prediction approach based on basis functions for time-varying frequency estimation of the frequency-modulated signal in a noisy environment is proposed.

The paper is organized as follows. We briefly describe the forward–backward prediction method for estimating the parameters of the autoregressive processes in Section 2. In Section 3, we propose a forward–backward basis function approach to estimate the signal time-varying frequency. The experimental results of estimating the time-varying frequencies are presented in Section 4. Concluding remarks are given in Section 5.

2. Forward–Backward Prediction for Estimation of Autoregressive Parameters

To derive the estimator of the autoregressive (AR) parameters suppose, that we are given the complex-valued signal $x(n),\ n = 1,\dots,N$, and let us consider the forward and backward linear prediction (LP) estimates of order p, given as Proakis *et al*. (1996)

$$\hat{x}(n) = -\sum_{k=1}^{p} a_k(n)x(n-k), \tag{1}$$

$$\hat{x}(n-p) = -\sum_{k=1}^{p} b_k(n)x(n+k-p). \tag{2}$$

The corresponding forward and backward errors are $h(n) = x(n) - \hat{x}(n)$, and $g(n) = x(n-p) - \hat{x}(n-p)$, where $a_k(n)$ and $b_k(n)$ are time-varying prediction coefficients in the forward and backward estimators, respectively. The method is based on the concept of forward and backward finite impulse response (FIR) filters of the signal $x(n)$

$$h(n) = \sum_{k=0}^{p} a_k(n)x(n-k) = \mathbf{x}^T(n)\mathbf{a}(n) = \mathbf{a}^T(n)\mathbf{x}(n), \tag{3}$$

$$g(n) = \sum_{k=0}^{p} b_k(n)x(n-p+k) = \mathbf{x}^T(n)\mathbf{J}\mathbf{b}(n) = \mathbf{b}^T(n)\mathbf{J}\mathbf{x}(n), \tag{4}$$

where $\mathbf{x}^T(n) = [x(n), x(n-1), \dots, x(n-p)]$, $\mathbf{a}^T(n) = [1, a_1(n), \dots, a_p(n)]$, $\mathbf{b}^T(n) = [1, b_1(n), \dots, b_p(n)]$, "^{T}" represents the matrix transposition transformation. $\mathbf{J}$ is an matrix with zeros except ones along the anti-diagonal and reverse in time a signal vector. The forward filter output $h(n)$ and backward filter output $g(n)$ depend on the $(p+1)$-dimensional vector $\mathbf{x}(n)$. Note that $a(0) = b(0) = 1$. We assume that $x(n)$ is available over the range $1 \leqslant n \leqslant N$ and FIR outputs can only be formed over the interval $p+1 \leqslant n \leqslant N$. It was shown in Marple (1987), Manolakis *et al.* (2005) that the forward and backward linear prediction parameters for a stationary random process are simply complex conjugates $a_k(n) = b_k^*(n)$, so the output $g(n)$ of the backward FIR filter in (4) may be expressed as

$$g(n) = \sum_{k=0}^{p} a_k^*(n)x(n-p+k) = \mathbf{x}^T(n)\mathbf{J}\mathbf{a}^*(n) = \mathbf{a}^H(n)\mathbf{J}\mathbf{x}(n), \tag{5}$$

where sign "*" means complex conjugate, and "^{H}" represents the matrix conjugate transposition operation.

For non-stationary signals, the forward and backward parameters are not time-invariant, and they may not be equal to the complex conjugate of each other. We consider, that $a_k(n) \cong b_k^*(n)$, and (5) is valid.

In contrast to the Burg approach (Kazlauskas, 2011; Zaknich, 2005), we use an unconstrained least-squares algorithm to determine the AR parameters. We form the forward and backward linear prediction estimates (1) and (2), and their corresponding forward and backward errors $h(n)$ and $g(n)$. Then, we combine both forward and backward variances normalized by $1/(2(N-p))$ to form the total variance estimate, which is

$$e = \frac{1}{2(N-p)} \sum_{n=p+1}^{N} \left[\left|h(n)\right|^2 + \left|g(n)\right|^2 \right]. \tag{6}$$

Since $|g(n)|^2 = |g^*(n)|^2$, then

$$e = \frac{1}{2(N-p)} \sum_{n=p+1}^{N} \left[\left|h(n)\right|^2 + \left|g^*(n)\right|^2 \right], \tag{7}$$

which is the same performance index as in Burg method. However, we do not impose the Levinson–Durbin recursion for the AR parameters. Substituting equations (3) and (4) into (7), we obtain

$$e = \frac{1}{2(N-p)} \sum_{n=p+1}^{N} \left[\left| \sum_{k=0}^{p} a_k(n)x(n-k) \right|^2 + \left| \sum_{k=0}^{p} a_k(n)x^*(n+k-p) \right|^2 \right]. \tag{8}$$

3. The Proposed Approach

The adaptive methods for time-varying AR parameter estimation may be used if the parameters of a signal change slowly. Adaptive systems are sensitive to noise level. We could reduce the sensitivity to the noise increasing the forgetting factor or step size of adaptive algorithms, but it decreases the convergence rate of the adaptive algorithms and ability of tracking the parameter change. Instead of using the adaptive algorithms for time-varying parameter estimation, we expressed $a_k(n)$ as a summation of constants multiplied by basis time functions (Niedzwiecki and Klaput, 2002).

$$a_k(n) = \sum_{i=0}^{m} a_{ki}\varphi_i(n), \quad n = 1, \ldots, N, \tag{9}$$

where a_{ki}, $k = 1, \ldots, p;\ i = 0, 1, \ldots, m$, are constants, m is the expansion dimension, and $\varphi_i(n)$ are basis time functions.

In the basis function approach not only the model order p, but also the basis functions $\varphi_i(n)$, and the expansion dimension m must be chosen. The time-varying parameters of a signal are changed to the summation of a set of unknown constants multiplied by predifined time-functions. The estimation by the basis function approach is to calculate not the time-varying parameters $a_k(n)$, but the unknown constant coefficients a_{ki}. The calculation is the same as for a stationary signal. If computational complexity and time are not restricted, the length of the block for the basis function approach can be of any length.

The functions $\varphi_i(n)$ must be independent and non-zero for $n = 0, 1, \ldots, N-1$, and $\varphi_i(n) = 1$, if $n = 0$. If a priori information about the signal variation is known, the basis functions should be chosen such that the trends in parameter change is retained. In case, when a priori information is unavailable, the basis function most commonly used are time polynomial, the Fourier, or cosine functions.

Substituting $a_k(n)$ from (9) into (8), we obtain

$$e = \frac{1}{2(N-p)} \sum_{n=p+1}^{N} \left\{ \left| \sum_{k=0}^{p} \sum_{i=0}^{m} a_{ki}\varphi_i(n)x(n-k) \right|^2 + \left| \sum_{k=0}^{p} \sum_{i=0}^{m} a_{ki}\varphi_i(n)x^*(n+k-p) \right|^2 \right\}. \tag{10}$$

Taking derivatives of e with respect to $a_{lj},\ l = 1, 2, \ldots, p;\ j = 0, 1, \ldots, m$, we have

$$\frac{\partial e}{\partial a_{lj}} = \frac{1}{(N-p)} \sum_{n=p+1}^{N} \left\{ \left[x(n) + \sum_{k=1}^{p} \sum_{i=0}^{m} a_{ki}\varphi_i(n)x(n-k) \right] \varphi_j^*(n)x^*(n-l) + \left[x^*(n-p) + \sum_{k=1}^{p} \sum_{i=0}^{m} a_{ki}\varphi_i(n)x^*(n+k-p) \right] \varphi_j^*(n)x(n+l-p) \right\}. \tag{11}$$

Equating (11) to zero, we get

$$\sum_{k=1}^{p}\sum_{i=0}^{m} a_{ki} \sum_{n=p+1}^{N} \big[\varphi_i(n)\varphi_j^*(n)x(n-k)x^*(n-l) + \varphi_i(n)\varphi_j^*(n)x(n+l-p)x^*(n+k-p)\big] = -\sum_{n=p+1}^{N} \big[\varphi_j^*(n)x(n)x^*(n-l) + \varphi_j^*(n)x^*(n-p)x(n+l-p)\big]. \quad (12)$$

Define

$$r_{ij}(k,l) = \sum_{n=p+1}^{N} \big[\varphi_i(n)\varphi_j^*(n)x(n-k)x^*(n-l) + \varphi_i(n)\varphi_j^*(n)x(n+l-p)x^*(n+k-p)\big]. \quad (13)$$

Then, (12) has the form

$$\sum_{k=1}^{p}\sum_{i=0}^{m} a_{ki} r_{ij}(k,l) = -r_{0j}(0,l), \quad l = 1,2,\ldots,p;\ j = 0,1,\ldots,m. \quad (14)$$

Equation (14) is a set of $p(m+1)$ linear equations and can be solved to get a_{ki}. After that, the time-varying coefficients $a_k(n)$ are computed using (9).

We can write (14) in a matrix form

$$R\,a = -h, \quad (15)$$

where $p(m+1) \times p(m+1)$ size matrix $R^T = \{\Psi(k,l)\}$, $k,l = 1,2,\ldots,p$, in which $(m+1)\times(m+1)$ size matrices $\Psi^T(k,l) = \{r_{ij}(k,l)\}$, $i,j = 0,1,\ldots,m$; $a^T = (a_1,\ldots,a_k,\ldots,a_p)$, in which $a_k = (a_{k0},\ldots,a_{ki},\ldots,a_{km})^T$ and $h^T = (\gamma_1,\ldots,\gamma_l,\ldots,\gamma_p)$, in which $\gamma_l = (r_{00}(0,l),\ldots,r_{0j}(0,l),\ldots,r_{0m}(0,l))^T$.

The direct or iterative methods are available for solving the matrix (15). In the direct methods some number of calculation steps are needed, which require a number of calculations of order q^3, where q is the size of matrix R. In case, when q is large, the number of computatations may be huge. The iterative methods calculates an approximations of the solution a. The iterations are stopped if a desired accuracy is achieved or a number of iterations are completed. The computatational complexity of iterative methods is of order q^2.

The algorithm computes the AR coefficients by (15). The time-varying frequency can be extracted from the time-varying autoregressive parameters $a_k(n)$. The nonstationary signal is modeled as the output of the time-varying AR process with a zero mean white noise input $w(t)$. From the estimates of AR parameters, we form the power spectrum estimate (Proakis and Manolakis, 1996)

$$P(f,n) = \frac{\sigma_w^2}{|1+\sum_{k=1}^{p} a_k(n)\exp(-j2\pi fk)|^2}, \tag{16}$$

where σ_w^2 is the variance of the white noise $w(n)$, f is the frequency. The variance of the white noise can be approximated by

$$\sigma_w^2 \approx \sigma_e^2 = \frac{1}{N-p}\sum_{n=p+1}^{N}\left[x(n)+\sum_{k=1}^{p} a_k(n)x(n-k)\right]^2, \tag{17}$$

where σ_e^2 is the variance of the parameter estimation error.

Time-varying frequencies can be found by estimating the peaks of the power spectral density $P(f,n)$. If there are several spectral peaks in the power spectral density function, a threshold must be set, and the peaks below this threshold belong to the noise. Another method to estimate the time-varying frequency is to form the estimation error filter polynomial $z^p + a_1(n)z^{p-1} + \cdots + a_p(n)$, and to calculate the roots $z_i(n)$. The frequency estimates are the angles of the roots

$$f_i(n) = \frac{1}{2\pi}\text{angle}\big(z_i(n)\big)f_s, \tag{18}$$

where f_s is the sampling frequency.

If the signal is real, then the roots are compex conjugate. In that case, the roots in the upper or lower half of a complex plane are selected.

4. Simulation Results

In this section we examine the performance of the proposed method and compare the results with that of the covariance algorithm. To investigate the ability of the proposed method, we generated a signal from the signal generator comprised of the sinusoid with time-varying frequency $f(n)$ embedded in a noise $w(n)$

$$x(n) = s(n) + w(n) = \cos\big(2\pi f(n)n\big) + w(n), \tag{19}$$

for $n = 1,\dots,N;\ w(n)$ is a zero-mean white Gaussian noise with unit variance $\sigma_w^2 = 1$. To get the desired signal-to-noise ratio (SNR) from the signal generator, the output signal is defined by

$$x(n) = s(n) + kw(n), \tag{20}$$

in which the coefficient k is computed such that

$$\text{SNR} = 10\log\frac{P_s}{k^2P_w}, \tag{21}$$

where $P_s = \frac{1}{N}\sum_{n=1}^{N} s^2(n)$ means the signal power, $P_w = \frac{1}{N}\sum_{n=1}^{N} w^2(n)$ is the noise power, and N is the length of the signal $s(n)$ and noise $w(n)$.

From (21) we obtain that for the desired SNR, coefficient k is calculated as follows

$$k = \frac{\sqrt{P_s}}{\sqrt{P_w}} 10^{-\frac{\mathrm{SNR}}{20}}. \tag{22}$$

We have used $L = 200$ Monte Carlo simulations in case of the additive noise. The same signals were used to demonstrate the superiority of the forward–backward basis function approach as compared with covariance algorithm.

We estimated the mean absolute frequency error (MAFE) as follows:

$$\mathrm{MAFE} = \frac{1}{N}\sum_{n=1}^{N} \left| f(n) - \hat{f}(n) \right|, \tag{23}$$

where $f(n)$ is the true frequency value at time n, and $\hat{f}(n)$ is the estimate of the true frequency value at time n

$$\hat{f}(n) = \frac{1}{N_r}\sum_{r=1}^{N_r} \hat{f}(n, r), \tag{24}$$

where $\hat{f}(n, r)$ is the estimate of the frequency at time n and at the rth experiment; N_r is the number of Monte-Carlo runs chosen here equal to 200.

The normalized power density spectrum (NPDS) estimate in dB is calculated according to the expression (25)

$$\mathrm{NPDS}(\hat{f}, n) = 10 \log \frac{P(\hat{f}, n)}{P_{\max}(\hat{f}, n)}. \tag{25}$$

By using the basis function approach, the time function $\varphi_i(n)$ and the expansion dimension m must be chosen. For the signal where frequency changes linearly , we have used the basis which consists of powers of the time $\varphi_i(n) = n^{i-1},\ i = 1, \ldots, m$ and $m = 2$. For the sinusoid with the periodically time-varying frequency, we have used the cosine function $\varphi_i(n) = \cos(\pi i n/N)$ with $m = 10$. The cosine function was also applied for the case of the frequency jump.

4.1. *Experimental Procedure*

To evaluate the approach described in Section 3, we have employed the proposed procedure as follows:

1. We have generated a signal with time-varying frequency $f(n)$ according to (19).
2. To obtain the desired SNR, we have calculated coefficient k according to (22).

3. We have expressed time-varying coefficients as a summation of constants multiplied by basis functions (9).
4. We have solved a set of (15), and have obtained a_{ki}.
5. We have computed the coefficients $a_k(n)$ according to (9).
6. Using the estimates of AR parameters, we have calculated the power spectrum estimates (16).
7. We have estimated the time-varying frequencies using (18).
8. We have estimated the mean absolute frequency error according to (23) and the normalized power density spectrum according to (25).

Table 1 illustrates the frequency error estimates averaged by $L = 200$ experiments and their confidence intervals $\Delta = \pm t_{\alpha/2;\ L-1}\frac{\hat{\sigma}}{\sqrt{L}}$, in which $\hat{\sigma}$ is the estimate of the standard deviation and α is the significance level. The value $t_{\alpha/2;L-1}$ is the point of Student's distribution with $L-1$ degrees of freedom which cuts the $\alpha/2$ part of the distribution. In case $\alpha = 0.05$ and $L = 200$, we find from Student's distribution table that $t_{0.025;\ 199} = 1.9720$.

In Table 1 there are some comparisons between the covariance method and the forward–backward basis function approach about the ability to estimate the time-varying frequency of two signals. The first signal is a unit amplitude sinusoidal signal with linearly time-varying frequency $f(n) = n$, and the second signal is a unit amplitude sinusoidal signal with periodically time-varying frequency $f(n) = 250 + 150\cos(2\pi n/150)$ in a noisy environment where SNR changes from 30 to -7 dB.

Figure 1 shows the instantaneous frequency estimates using the forward–backward basis function approach where the frequency changes linearly (A), periodically (C), and have two jumps (E). In (B), (D), and (F) are absolute differencies between true and estimated instantaneous frequencies in noisy $\mathrm{SNR} = 0$ environment, in case where model order p is equal to 4. As we can see in Fig. 1, algorithm can track the changes of the true frequency.

Figure 2 shows the comparison of the covariance method with forward–backward basis function approach when frequency changes periodically (see Table 1). Model order is equal to 15, expansion dimensijon is equal to 10, and SNR changes from -8 to 10 dB. MAFE of the proposed approach is smaller as compared with the covariance method, especially in the $[-8, 0]$ dB interval.

Remarks. The accuracy of the estimated frequency with time depends on signal length n: if $n \geqslant 70$, MAFE between true and estimated frequencies are less as compared with the case, when $n < 70$ (see Fig. 1). The MAFE of estimated frequency also depends on SNR: for example, if $\mathrm{SNR} = 10$, then $\mathrm{MAFE} = 1.0746$, and if $\mathrm{SNR} = -3$, then $\mathrm{MAFE} = 3.7101$ and so on (see Table 1). Finally, the MAFE depends on method: for example, if $\mathrm{SNR} = 0$, for covariance method $\mathrm{MAFE} = 4.3465$, and for forward–backward approach $\mathrm{MAFE} = 1.5418$ (see Table 1).

Table 1

Mean absolute frequency errors (MAFE) and confidence intervals Δ of the covariance method and the forward–backward basis function approach where frequency $f(n)$ changes linearly and periodically. SNR is the signal-to-noise ratio; p is the order of the predictive filter. Signal sampling rate f_s is 1000 Hz.; n is the signal length. Monte Carlo runs are equal to 200

SNR (dB)	Mean absolute frequency estimation error (Hz)		Notes
	Covariance method	Forward–backward approach	
30	$0.9799 \pm 7.958 \times 10^{-5}$	$0.9683 \pm 6.878 \times 10^{-5}$	$f(n) = n$,
20	1.0028 ± 0.0010	0.9991 ± 0.0007	$p = 4$,
10	1.7561 ± 0.0123	1.7146 ± 0.0106	$n = [30, 400]$
5	3.8560 ± 0.0204	3.8231 ± 0.0181	
0	7.6240 ± 0.0392	7.4163 ± 0.0364	
− 3	13.1325 ± 0.1608	12.1841 ± 0.1232	
− 5	21.1101 ± 0.2310	20.5430 ± 0.3102	
− 7	38.7370 ± 0.3766	37.2391 ± 0.3456	
30	1.3224 ± 0.0516	1.0240 ± 0.0231	$f(n) = n$,
20	1.4487 ± 0.0565	0.9804 ± 0.0012	$p = 12$,
10	1.8585 ± 0.0884	1.0746 ± 0.0285	$n = [30, 400]$
5	2.5586 ± 0.1235	1.1510 ± 0.0306	
0	4.3465 ± 0.1409	1.5418 ± 0.0426	
− 3	7.8959 ± 0.1732	3.7101 ± 0.1265	
− 5	14.1029 ± 0.3134	9.1688 ± 0.2176	
− 7	24.3140 ± 0.4021	19.5582 ± 0.3376	
30	$0.8897 \pm 1.9965 \times 10^{-5}$	$0.8887 \pm 1.4900 \times 10^{-5}$	$f(n) = 250+$
20	$0.8946 \pm 2.7760 \times 10^{-4}$	$0.8923 \pm 2.5757 \times 10^{-4}$	$150 \cos(2\pi n/150)$
10	1.1299 ± 0.0032	1.1096 ± 0.0025	$p = 4$,
5	2.3149 ± 0.0069	2.2149 ± 0.0067	$n = [30, 400]$
0	5.5340 ± 0.0351	5.4131 ± 0.0289	
− 3	10.3270 ± 0.1063	9.4521 ± 0.0903	
− 5	17.2110 ± 0.1770	16.0501 ± 0.1631	
− 7	32.5211 ± 0.3581	30.6514 ± 0.2870	
30	1.2110 ± 0.0493	0.9415 ± 0.0163	$f(n) = 250+$
20	1.2530 ± 0.0471	0.9540 ± 0.0242150	$\cos(2\pi n/150)$
10	1.5161 ± 0.0608	0.9272 ± 0.0113	$p = 12$,
5	1.8160 ± 0.0613	1.1040 ± 0.0318	$n = [30, 400]$
0	2.9661 ± 0.0810	1.3996 ± 0.0298	
− 3	5.6362 ± 0.1380	3.1611 ± 0.0879	
− 5	10.0270 ± 0.2036	6.6560 ± 0.1664	
− 7	19.7380 ± 0.2335	16.2961 ± 0.2760	
30	3.2274 ± 0.1180	1.5550 ± 0.0598	$f(n) = 250+$
20	3.6661 ± 0.1136	1.5530 ± 0.0632150	$\cos(2\pi n/150)$
10	4.4060 ± 0.1345	1.6261 ± 0.0641	$p = 20$,
5	5.3911 ± 0.1452	1.7160 ± 0.0611	$n = [30, 400]$
0	7.1910 ± 0.1633	2.4430 ± 0.0777	
− 3	10.4280 ± 0.1911	3.2440 ± 0.0999	
− 5	13.3150 ± 0.2189	6.7810 ± 0.1564	
− 7	21.2340 ± 0.2727	13.1080 ± 0.2048	

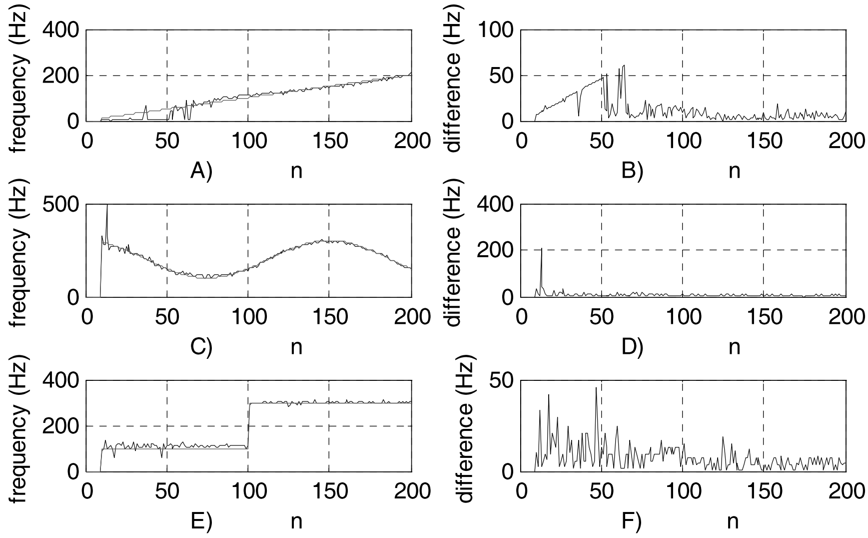

Fig. 1. Time-varying frequency estimates using the forward–backward basis function approach. (A) Instantaneous frequency estimates where frequency changes linearly; (C) Instantaneous frequency estimates where frequency changes periodically; (E) Instantaneous frequency estimates where frequency changes have two jumps. In Figs. (B), (D), (F) are absoliute differencies between true and estimated instantaneous frequencies. Sampling frequency of the signals $f_s = 1000$ Hz, signal-to-noise level $\mathrm{SNR} = 0$ dB, model order $p = 4$; Monte Carlo runs are equal to 200.

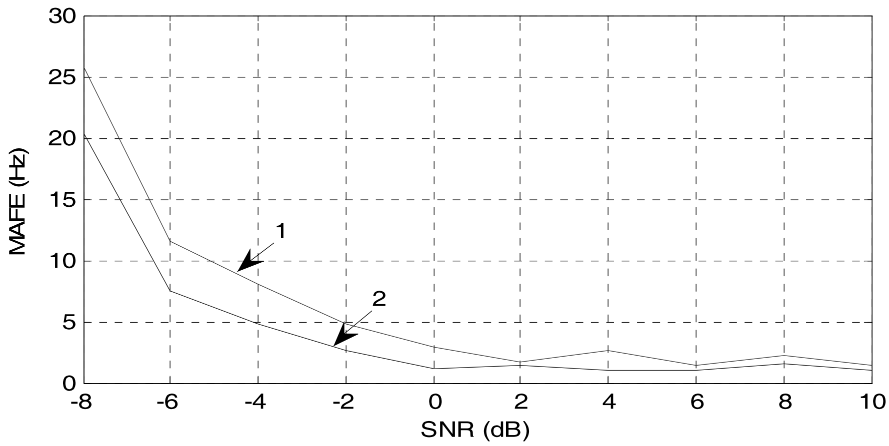

Fig. 2. Results for comparative study between the covariance method (1) and the proposed forward–backward basis function approach (2). MAFE versus SNR: sampling frequency $f_s = 1000$ Hz; model order $p = 15$; expansion dimension $m = 10$. Frequency changes periodically (see Table 1). Monte-Carlo runs are equal to 200.

5. Conclusions

In the paper the approach has been developed to estimate the rate of frequency change of a nonstationary signals with time-varying frequency. The time-varying frequencies of the nonstationary signals are contained in the time-varying coefficients of the autoregressive model. Extracting the frequency information consists of two steps, first, the time-varying AR parameter estimation at time n, and then the frequency calculation. We used the basis function approach utilizing the explicit model for the parameter variation. The basis function approach, in which the time-varying parameters are expanded as a summation of the weighted time functions are capable of tracking both the fast and the slow time-varying frequencies. The selection of the expansion dimension and the basis function depend on the character of the frequency variation. The novelty of this approach is that it presents a simple method for estimating a time-varying frequency derived from a forward–backward prediction technique where time-varying parameters are expressed as a summation of constants multiplied by basis functions. The simulation results confirm the theoretical analysis and show the potential of the new approach in a noisy environment.

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Dažniu moduliuoto signalo stebimo triukšmuose momentinio dažnio įvertinimas panaudojant tiesioginės ir atgalinės prognozės metodą

Kazys KAZLAUSKAS, Rimantas PUPEIKIS

Straipsnyje pasiūlytas signalo stebimo triukšmuose momentinio dažnio įvertinimas panaudojant tiesioginės ir atgalinės prognozės bazinių funkcijų metodą. Pirmiausia tiesioginės ir atgalinės prognozės metodu įvertinami signalo autoregresijos modelio kintami laike parametrai, kurie aprašomi konstantų ir bazinių funkcijų sandaugų suma. Po to pagal įvertintus signalo parametrus apskaičiuojami momentiniai dažniai. Eksperimento rezultatai parodė, kad šis metodas yra pranašesnis už tiesioginės prognozės metodą.

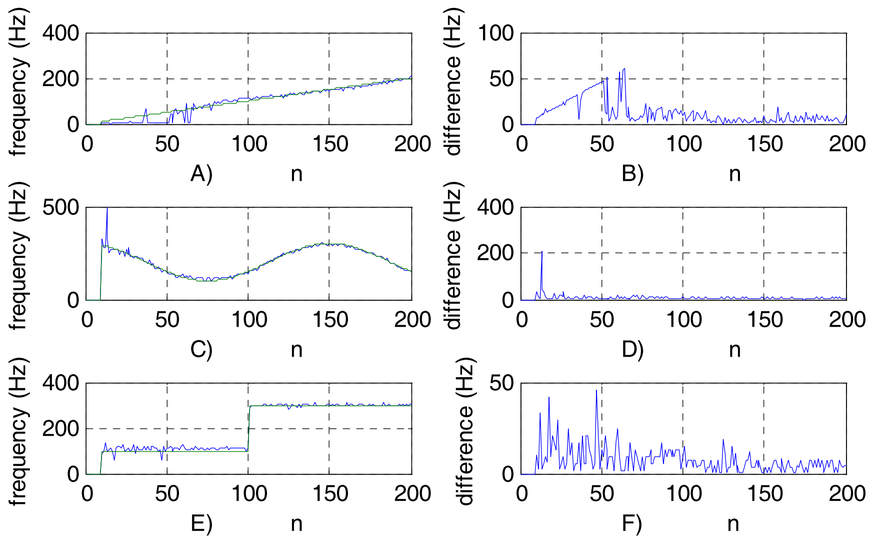

Fig. 1. Time-varying frequency estimates using the forward–backward basis function approach. (A) Instantaneous frequency estimates where frequency changes linearly; (C) Instantaneous frequency estimates where frequency changes periodically; (E) Instantaneous frequency estimates where frequency changes have two jumps. In Figs. (B), (D), (F) are absoliute differencies between true and estimated instantaneous frequencies. Sampling frequency of the signals $f_s = 1000$ Hz, signal-to-noise level $\mathrm{SNR} = 0$ dB, model order $p = 4$; Monte Carlo runs are equal to 200.

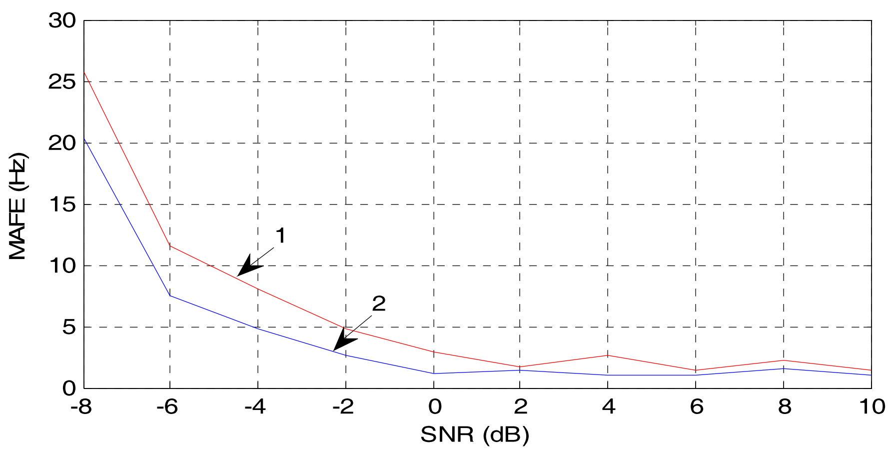

Fig. 2. Results for comparative study between the covariance method (1) and the proposed forward–backward basis function approach (2). MAFE versus SNR: sampling frequency $f_s = 1000$ Hz; model order $p = 15$; expansion dimension $m = 10$. Frequency changes periodically (see Table 1). Monte-Carlo runs are equal to 200.